

# Remarks on Statistical Properties of the Turbulent Interstellar Medium

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**Abstract.** The supernova-driven interstellar medium in star-forming galaxies has Reynolds numbers of the order of  $10^6$  or even larger. We study, by means of adaptive mesh refinement hydro- and magnetohydrodynamical simulations that cover the full available range (from 10 kpc to sub-parsec) scales, the statistical properties of the turbulent interstellar gas and the dimension of the most dissipative structures. The scalings of the structure functions are consistent with a log-Poisson statistics of supersonic turbulence where energy is dissipated mainly through shocks.

**Keywords.** Turbulence, magnetohydrodynamics, ISM: general, ISM: structure

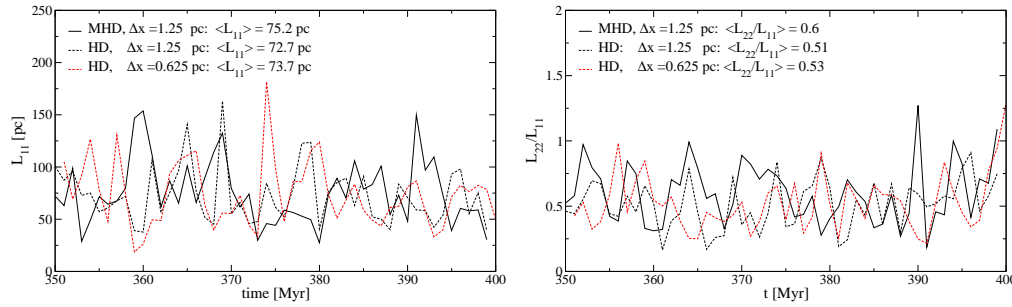
## 1. Introduction

The interstellar medium is a highly turbulent compressible flow with Reynolds numbers of the order of  $10^6$  or even larger (e.g., Elmegreen & Scalo 2005). Supernovae are the main source of energy of the interstellar gas, which is dominated by nonlinear processes, including heating and cooling, that transfer energy among a wide range of scales. It is unclear what are the driving scales in the interstellar medium as observations show different values ranging from a few parsec to tens or even hundreds of parsec (see, e.g., Haverkorn et al. 2005). Furthermore, it is still under debate which dissipation process dominates the interstellar turbulence. High-resolution simulations of the interstellar medium that cover a wide range of scales, including the injection and viscous scales, are valuable tools to address these issues. We, therefore, carried out kpc-scale high resolution (up to 0.625 pc) hydro- and magnetohydrodynamical simulations of the supernova-driven interstellar medium (Avillez & Breitschwerdt 2004, 2005, 2006) allowing us to tackle simultaneously the disk and halo evolution and to understand the issues referred above. In this paper we briefly discuss these results. Section 2 deals with the injection scales, the scalings of the velocity structure functions and dissipation processes are referred to in section 3, followed by a discussion of the results and their implications (section 4).

## 2. The Injection Scale of Interstellar Turbulence.

The outer scale of the turbulent flow in the ISM is related to the scale at which the energy in blast waves is transferred to the interstellar gas. Such a scale can be determined by using the so-called two-point correlation function  $R_{ij}(\vec{l}, t) = \langle u_i(\vec{x} + \vec{l}, t) u_j(\vec{x}, t) \rangle$ , where  $u_i$  are the components of the fluctuating velocity field  $\vec{u}$ . The diagonal components of  $R_{ij}$  are even functions of  $\vec{l}$  and can be written in terms of dimensionless scalar functions  $f(l, t)$  and  $g(l, t)$ , which satisfy  $f(0) = g(0) = 1$  and  $f, g \leq 1$ , that is,  $R_{11}/u^2 = f(l, t)$  and  $R_{ij}/u^2 = g(l, t)$  if  $i = j \neq 1$  and zero if  $i \neq j$  with  $u = (\frac{1}{3}\langle \vec{u}^2 \rangle)^{1/2}$ . In these equations  $R_{11}$

and  $R_{22} = R_{33}$  are known as the longitudinal and transverse autocorrelation functions, respectively.



**Figure 1.** History of the characteristic size (given by  $L_{11}$ ) of the larger eddies (left panel) and of the ratio  $L_{22}/L_{11}$  (right panel) for the HD (dashed lines) and MHD (solid line) runs for 1.25 pc (black) and 0.625 pc (red) resolutions.

The outer scale of the flow is given by  $L_{11} = \int_0^{+\infty} f(l, t) dl$ , which is calculated in a region with a linear size of 500 pc at a distance of 250 pc from the edges of the computational domain such that we avoid the periodicity effects of the boundary conditions. The top panel of Figure 1 shows the history of  $L_{11}$  in the last 50 Myr of evolution of the simulated interstellar medium for 1.25 (in black) and 0.625 pc (in red) resolutions. The average integral length scale in the three simulations is 73–75 pc and seems independent of the resolution. In all the cases there is a large scatter of  $L_{11}$  around its mean as a result of oscillations in the local Galactic star formation rate, where the formation and merging of superbubbles, is responsible for the peaks observed in the two plots. The correlation length scale measured in the simulations is remarkably similar to that determined by Kaplan (1958) who found that interstellar velocities start to decorrelate at a scale of 80 pc.

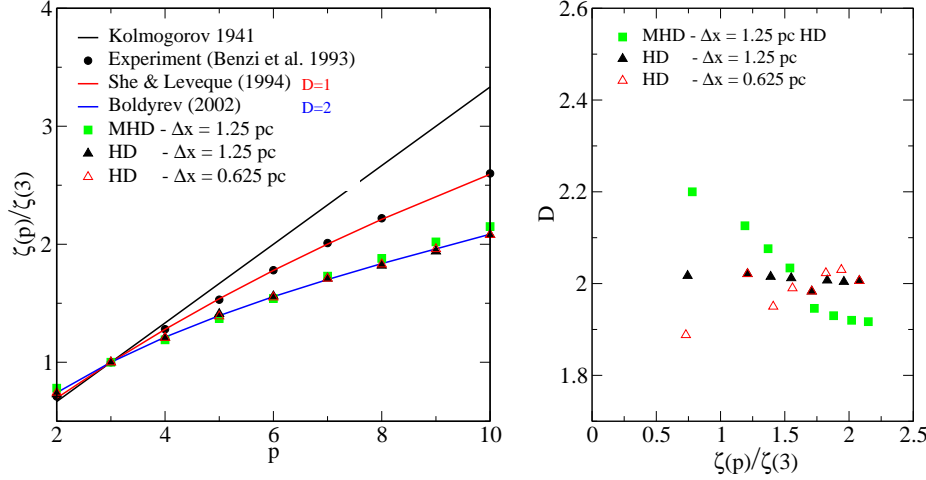
The transverse integral length scale given by  $L_{22} = \int_0^{+\infty} g(l, t) dr$  equals  $0.5L_{11}$  in the case of isotropic turbulence. In the present simulations we have  $0.2 < L_{22}/L_{11} < 1.3$  (bottom panel of Figure 1). In spite of the large scatter the time average of the  $L_{22}/L_{11}$  over a 50 Myr period is 0.51 and 0.6 for the HD and MHD runs, respectively. The discrepancy of  $\langle L_{22}/L_{11} \rangle \sim 0.5$  by about 20% in the MHD run is a consequence of the anisotropy in the flow introduced by the magnetic field. Overall the field strength is still too low to produce a larger deviation. In case of the HD run  $\langle L_{22}/L_{11} \rangle \sim 0.5$ , indicates that in a statistical sense the interstellar unmagnetised turbulence is roughly isotropic.

### 3. Velocity Structure Functions and the Most Dissipative Structures

Turbulent flows are usually described in a statistical fashion by the velocity structure functions of order  $p$  defined as  $S_p(l) = \langle \delta v_l^p \rangle \propto l^{\zeta(p)}$ , where  $\zeta(p)$  is a scalar,  $\delta v_l = |v(x+l) - v(x)|$  with  $v(x+l)$  and  $v(x)$  being the velocities along the  $x$ -axis at two points separated by a distance  $l$  such  $\eta \ll l \ll L_{11}$ , with  $L_{11}$  being the energy integral scale and  $\eta = \nu^{3/4} \epsilon^{-1/4}$  the Kolmogorov microscale, respectively. Here,  $\nu$  is the kinematic viscosity and  $\langle \rangle$  stands for the ensemble average over the probability density function of  $\delta v_l$ . From the Kolmogorov (1941; hereafter denoted by K41) theory one derives

$$\langle \delta v_l^p \rangle \propto \langle \delta v_l^3 \rangle^{\zeta(p)/\zeta(3)}. \quad (3.1)$$

with  $l \gg \eta$ . This property, referred to as extended self-similarity (ESS), indicates that  $S_p$  is dictated solely by the third-order structure function  $S_3$  via the set of scalings  $\zeta(p)/\zeta(3)$  and is valid in a wide range of length scales for large as well as small Reynolds numbers even if no inertial range is established (Benzi et al. 1993). Using the ESS concept



**Figure 2.** *Left:* Exponent  $\zeta(p)/\zeta(3)$  for the structure function versus order  $p$ . The full line corresponds to  $\zeta(p) = p/3$  (K41 theory), bullets correspond to data of Benzi et al. (1993), red and blue lines refer to the She-Levéque (1994) and Burgers-Kolmogorov (Boldyrev 2002) models, respectively. Triangles represent data from HD simulations with 1.25 pc (black) and 0.625 pc (red) resolutions and the green squares refer to the MHD run with 1.25 pc resolution. *Right:* Fractal dimension  $D$  of the most dissipative structures (derived from eq. 3.2) as function of the exponent  $\zeta(p)/\zeta(3)$ , with  $p = 2, 4, \dots, 10$ , shown in the left panel for the three simulations.

we determined the velocity structure functions  $\langle \delta v_l^p \rangle$ , with order  $p = 2, 4, \dots, 10$  as a function of  $\langle \delta v_l^3 \rangle$  and their best fits slopes  $\zeta(p)/\zeta(3)$  (see Avillez & Breitschwerdt 2006) for the simulated interstellar gas. The slopes are displayed as function of the order  $p$  in the left panel of Figure 2, where triangles (black corresponds to  $\Delta x = 1.25$  pc and red to  $\Delta x = 0.625$  pc) and squares refer to the HD and MHD runs, respectively.

From these slopes one can determine the Hausdorff dimension of the most dissipative structures through the solution of (Dubrulle 1994)

$$\frac{\zeta(p)}{\zeta(3)} = \Theta(1 - \Delta)p + \frac{\Delta}{1 - \beta} (1 - \beta^{\Theta p}), \quad (3.2)$$

where  $\Delta = 1 - \Theta$  is a parameter depending on the Hausdorff dimension  $D$  of the most dissipative structures,  $\beta = 1 - \Delta/(3 - D)$  is a measure of the intermittency and  $\Theta$  is a parameter that depends on cascade of the energy transfer in the inertial range. If this cascade is Kolmogorov, then  $\Theta = 1/3$ . The above mentioned ratios correspond to  $D = 1.9 - 2.02$  (right panel of Figure 2), implying that the energy injected by supernovae into interstellar turbulence is dissipated preferentially through shocks (i.e., 2D surfaces).

Further analysis on the Hausdorff dimension of the most dissipative structures can be drawn from the comparison of the simulated and theoretical predictions (eq. 3.2) of the  $\zeta(p)/\zeta(3)$  variation with  $p$  shown in Figure 2 (left panel). For comparison we also show the variation of the ratio  $\zeta(p)/\zeta(3)$  observed experimentally by Benzi et al. (1993) and predicted by the K41 and She-Levéque (1994; hereafter denoted by SL94) models in incompressible turbulence and the Burgers-Kolmogorov model (Boldyrev 2002; hereafter

denoted by BK02) for supersonic turbulence. The K41 model considers a log-normal statistics for the transfer of energy from large to small scales and has no corrections to intermittency. The SL94 and BK02 models use a log-Poisson statistics (see discussion in Dubrulle 1994) to describe intermittency.

The figure shows that there exists a small discrepancy between the MHD data (green squares) and the BK02 curve and a decrease of  $D$  with increase of  $p$ . This variation, although within the errors of numerical noise, indicates a tendency toward filamentary dissipative structures due to the anisotropy induced by the magnetic field.

#### 4. Discussion and Final Remarks

The dissipation of energy in interstellar turbulence proceeds through shocks and the variation of  $\zeta(p)/\zeta(3)$  with  $p$  is most consistent with a log-Poisson model where at the inertial range the cascading of energy behaves more like Kolmogorov. Similar scalings have been found by Boldyrev et al. (2002) and Padoan et al. (2004) for the dissipation of energy in molecular clouds. Note, however, that our simulations include molecular clouds and its fragmentation and cover a larger range of scales (from kpc to 0.6 pc) than those used by these authors. The simulations also show that only a small amount of energy is dissipated through shocks in the inertial range and the energy decay follows a quasi-Kolmogorov spectrum. However, the shock structures start to play an important rôle in energy transfer and dissipation near the small scales.

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#### References

- Avillez, M. A., & Breitschwerdt, D. 2004, *A&A*, 425, 899
- Avillez, M. A., & Breitschwerdt, D. 2005, *A&A*, 436, 585
- Avillez, M. A., & Breitschwerdt, D. 2006, *Phys. Rev. Lett.*, submitted
- Benzi, R., Ciliberto, S., Tripiccone, R., Baudet, C., Massaioli, F., & Succi, S. 1993, *Phys. Rev. E* 48, R29
- Boldyrev, S. 2002, *ApJ* 569, 841
- Boldyrev, S., Nordlund, Å., & Padoan, P. 2002, *Phys. Rev. Lett.* 89, 31102
- Dubrulle, B. 1994, *Phys. Rev. Lett.* 73, 959
- Elmegreen, B., & Scalo, J. 2004, *ARA&A* 42, 211
- Haverkorn, M., Gaensler, B. M., Brown, J. C., Bizunok, N. S., McClure-Griffiths, N. M., Dickey, J. M., & Green, A. J. 2006, *ApJ* 637, L33
- Kaplan, S. A. 1958, in "Electromagnetic Phenomena in Cosmical Physics", Proceedings of IAU Symposium no. 6, ed. B. Lehnert, Cambridge University Press, p. 504
- Kolmogorov, A. N. 1941, *C.R. Acad. Sci. URSS* 30, 301
- Padoan, P., Jimenez, R., Nordlund, Å. & Boldyrev, S. 2004, *Phys. Rev. Lett.* 92, 1102
- She, Z.-S., & Lévéque, E. 1994, *Phys. Rev. Lett.* 72, 336